For an excellent grade you have to get minimum of 9 questions accepted of each set, for a 'good' - at least 7, and for 'passed' - 5 questions.

Group definitions, morphisms, permutations group

Problem 1.1: Show, that any permutation can be represented

- as a product of permutations of the form $(1, i)$;
- as a product of permutations of the form $(i, i + 1)$.

Defenition 1.1. $\mathrm{sign}(\pi) = (-1)^t$*, where t is the amount of transpositions in π*

Problem 1.2: Show that sign doesn't depend on product decomposition. Formulate the definition of sign based on amount of inversion in permutation.
Problem 1.3: Show that $\mathrm{sign} : S_n \to \mathbb{Z}_2$ is an homomorphism
Problem 1.4: Prove that the following are isomorphism:

$$S_3 \simeq \text{[Equilateral triangle symmetry group]}$$
$$S_4 \simeq \text{[Cube rotations group]}$$
$$S_4 \times S_2 \simeq \text{[Cube symmetry group]}$$

Problem 1.5: Prove that the group of 6 element is either abelian or isomorphic to S_3

Defenition 1.2. *Let f be isomorphism $G \to G$. Then f is called an automorphism of G. Group of some group's G automorphisms is denoted as* $\mathrm{Aut}\, G$*.*

Problem 1.6: Show, that $\mathrm{Aut}(\mathbb{Z}_p) \simeq (\mathbb{Z}_p)$
Problem 1.7: Show that any infinite group has a non-trivial subgroup.
Problem 1.8: Show that any group of order 8 has the following form: $\{1, a, a^2, a^3, b, ab, a^2b, a^3b\}$.
Problem 1.9: Which is greater: number of elements of S_n of odd order or of even order?
Problem 1.10: Let G be a set with a following binary operation $/$:

$$G \times G \to G : (g, h) \mapsto g/h$$

$$\forall f, g, h \in G : (f/h)/(g/h) = f/g$$
$$\forall g, h \in G, \exists x \in G : g/x = h$$

Show, that G is group with respect to the following product: $gh = g/((h/h)/h)$.