For an excellent grade you have to get minimum of 9 questions accepted of each set, for a 'good' - at least 7, and for 'passed' - 5 questions.

Groups SU(2), SO(3)

Problem 3.1: Show:

$$e^{xA}Be^{-xA} = \sum \frac{x^k}{x!} \operatorname{ad}_A^k(B),$$

where ad is defined as: $\operatorname{ad}_a(b) = [a, b]$.

Problem 3.2: Show that center of $SO(n)$ either trivial, or has order 2. What is the center of $SU(n)$?.

Problem 3.3: Let us introduce metric on $SU(2)$:

$$ds^2 = \operatorname{Tr}(dg \cdot dg^\dagger)$$

Show that it corresponds with standard sphere metric.

Problem 3.4: Show that any element $O \in SO(3)$ has an eigenvector with eigenvalue 1.

Problem 3.5: Show that Lie Algebra $\mathfrak{so}(3)$ is a vector space of antisymmetric matrices.

Let H be the space of traceless hermitian 2x2 matrices:

$$H = \{h \in \mathbb{C}^{2x2} | \operatorname{Tr}(h) = 0, h = h^\dagger\}$$

Problem 3.6: Show that Pauli matrices (σ_i) forms a basis for H. Show that $\forall U \in SU(2), \forall m \in H :$ $U^\dagger m U = n \in H$

Problem 3.7: Let $m = m_i\sigma_i \in H$. Show that

$$\omega : SU(2) \to \operatorname{Aut}(\mathbb{R}^3)$$

$$U \mapsto \omega(U)$$

such that $n_i = \omega(U)_{ij} m_j$can be represented as

$$\omega(U)_{ij} = \frac{1}{2} \operatorname{Tr}(\sigma_i U^\dagger \sigma_j U)$$

Show that it is an homomorphism.

Problem 3.8: Show that $\omega(U) \in SO(3)$.

Problem 3.9: Show that $SO(3) \simeq SU(2)/Z_2$